DESIGN OF EXPERIMENTS AND SIX SIGMA METHODS IN LOGISTICS

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Abstract: More and more factors have an influence on effectiveness and efficiency in logistical processes and systems. An important question of science is to identify the most important factors of control complex logistical processes and systems, to know their levels and influences to control all things in a right manner. To find the optimum in control there are often a lot of experiments to realize – practical and theoretical ones. In this field are used the sensitivity analysis as well as simulation and the Design of Experiments (DoE) too. This is necessary to fast prevent failures and solve problems early. This task is not an easy one, especially in logistical networks where a failure can have a lot of causes. In the following some of the connections between the both sciences Quality Management methods and Logistics are discussed and examples are given to show the synergy and the greater effects which make the synergy effect possible. (Compare [1] [2]) Two special areas of methods are focused in this paper: Methods of DoE and of Six Sigma.

Keywords: Logistics, DoE, Design of Experiments, Six Sigma, Quality Management, methods

1. Quality Management methods in the Logistics area

The use of the well known QM-methods makes it possible to recognize failures and their causes in order to analyze logistical processes and systems (Fig. 1).

![Methods, processes and techniques of failure analysis](image)

Figure 1. Overview of some important quality management methods [3]
The methods with a preventive character can be used to create new processes and systems or to optimise existing ones. At first it should be asked for the requirements of the customers to define the goals of the logistical services (QFD). The FMEA was successfully used at many companies. The quality control charts are also widely used with a great success [4]. The preventive methods can be effectively used in addition to the analytical ones [5] [6]. Statistical Process Research including the Design of Experiments (DoE) are one of them.

2. Design of Experiments – goals and Method

DoE has an old tradition and history by R. Fisher, Taguchi, Shainin i.e. Some examples for using Shainin methods in Logistics are given in table 1.

Table 1. Examples for using SHAININ-Methods in Logistics

<table>
<thead>
<tr>
<th>SHAININ methods</th>
<th>Examples for using SHAININ-Methods in Logistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Multi-vary-chart</td>
<td>Failures in packaging or commissioning</td>
</tr>
<tr>
<td>2 Component search</td>
<td>Analysis of cargo units</td>
</tr>
<tr>
<td>3 Paired comparisons</td>
<td>Compare two high rack facilities</td>
</tr>
<tr>
<td>4 Variables search</td>
<td>Prevent failures in packaging by changing the design of packing, the conveying means or the person who fulfil the task</td>
</tr>
<tr>
<td>5 Full factorials</td>
<td>Maximize the rate of identified objects in automatically logistical systems</td>
</tr>
<tr>
<td>6 B vs C (Better vs Current)</td>
<td>New design of cargos</td>
</tr>
<tr>
<td>7 Scatter plots</td>
<td>Trouble-shooting in commissioning</td>
</tr>
</tbody>
</table>

Design of Experiments (DoE) is a structured, an established and a organized method of quality management (Comp. Fig.1). The key factor to minimize optimisation costs is to realize as few experiments as possible. DoE requires only a small set of experiments and thus helps to reduce costs. But and that is a surprise, DoE is only seldom used in the logistics area because it is real unknown in this field. The Shainin (Table 1) methods include the full factorial ones. These are typical methods of the classical statistical research methods. DoE is used to determine the relationship between the different factors (x_i) affecting a process and the output (result) of that process (y). DoE has more to offer than a “one change at a time experimental method”. “One change at a time experimental method” has always the risk that the researcher find only the significant effect on the output. DoE also focuses on dependency and interaction between the factors. DoE plans for all possible dependencies at first, and then prescribes exactly which data are needed to assess them. The exact length and size of the experiment are set by the experimental design before the real experiments are beginning. Design of Experiments involves designing a set of experiments, in which all relevant factors are varied systematically (Fig. 2). When the results of these experiments are analysed, they help to identify optimal conditions. Further results of DoE are:
- The factors that most influence the results (high effect).
- The factors that little influence the results (small effect).
- The existence of interactions and synergies between factors.

The common way to use DoE is the following:
1. Define the objective of the investigation.
2. Define the variables (factors) y_k that will be measured to describe the output.
3. Define the variables (factors) that will be controlled x_i during the experiment.
4. Define the ranges of variation and the factor levels of each factor.
5. Define and optimise the experimental plan.
6. Prepare and carry out the experiments carefully and secure the results.
7. Do the statistical analysis and interpret the results.
8. Use the knowledge to optimise the process or system.

This is only a short introduction in DoE. Use the literature in [7] and [8] for further information about the basics of DoE.

3. Design of Experiments (DoE) in Logistics

Now is given a small example how to use DoE in the logistics area:

**Problem and goal:** A sorter is a complex logistical technical system which includes the whole logistical sorting process too. The task of this sorter is to put small parcels, packages, fragile and sensitive products as well as heavy freight goods, boxes and baggage safety onto the conveyor. After that it is necessary to discharge them into the right destination. The main goal is to sort all goods in a fast manner to achieve a high through put rate, a high efficiency of the sorter process and a short through put time of the goods too. Shortly mentioned, 100 percent of goods should be sorted at the first time. To calculate this real rate of failures the number of parcels and other objects that have more than one tour in the sorter circle are used. On one side the goal is defined to minimize the number of parcels that have more than one sorter circulation necessary to achieve the right destination. On the other side it should be sorted as much goods as possible.

**Design of the Experiments:** The engineers believe that there are lot of factors influence the problem, but there are only two important ones that have a big influence on the result:
- The speed of the sorter (factor 1) and
- The pitch between the parcels (factor 2).

That is why there is given a two factor problem with $k = 2$ factors.

The engineers know that a low speed is good and a small pitch is good too. But what is about the rate of failures? Will the rate of failures increase due to the speed and due to a greater pitch? The experiments should give an answer to the following questions:
- How many experiments are necessary to analyse the problem?
- How many failures will appear at each combination?
- How big is the influence of speed and of pitch on the rate of failures?
- Is there any significant interaction between both factors – speed and pitch?
- Which combination of speed and pitch will have the lowest rate of failures?
- Are there limits of the sorter visible?
- How much failures should be accepted?

Next step is to determine the recommended factor levels as shown in the following (Table 2):

| Table 2. Factor levels of the factors speed and pitch |
|--------------------------|----------------|----------------|
| $x_1$ speed (m/s)        | $5$            | $10$           |
| $x_2$ pitch (m)          | $0.2$          | $0.4$          |
Each factor gets two factor levels, a low one and a high one. The low one get the value of (-1), the high one get a value of (+1). There are two factor levels with p = 2. That allows four combinations (m = 4) due to formula 1

\[ m = k^p = 2^2 = 4, \]  

where

- \( m \) = combinations; \( k \) = factors; \( p \) = factor levels.

The task is now to get more information about the question which combination will be the best. To make a statistical analysis two tests or more which each combination are necessary to calculate statistical parameters. Each combination will be realized three times. The number of replications n is three. (Sometimes two replications are enough!). If there will done three tests three results \((y_1, y_2,\text{ and } y_3)\) for each combination will be achieved. The whole number of experiments \( N \) is calculated as following (Formula 2):

\[ N = m \times n = 4 \times 3 = 12 \]  

where

- \( N \) = experiments; \( m \) = combinations; \( n \) = replications.

Because the goods have a high variety there are used predefined test charges of goods. Then the arithmetic mean of all these test charges is calculated. 5.4 is the arithmetic mean in the first test of the first combination over all test charges of goods at all. The following results (Table 3) are achieved with three replications of each combination. It is important to mention that the sequence of the tests should be full randomised! The statistical parameters that have to be calculated at first are \( Y \) = the arithmetic mean of the results and \( s_i^2 \) = the variance of the results.

Table 3. Results and first statistical parameters of the experiments

<table>
<thead>
<tr>
<th>Number of combination</th>
<th>x₁</th>
<th>x₂</th>
<th>x₁x₂</th>
<th>y₁</th>
<th>y₂</th>
<th>y₃</th>
<th>Y</th>
<th>( s_i^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>8.3</td>
<td>8.6</td>
<td>9.1</td>
<td>8.7</td>
<td>0.163</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>15.1</td>
<td>15.4</td>
<td>15.9</td>
<td>15.5</td>
<td>0.163</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>5.4</td>
<td>4.2</td>
<td>4.4</td>
<td>4.7</td>
<td>0.413</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10.2</td>
<td>11.0</td>
<td>11.4</td>
<td>10.9</td>
<td>0.373</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td></td>
<td></td>
<td>39.67</td>
<td>39.67</td>
<td>39.67</td>
<td>39.67</td>
<td>1.11</td>
</tr>
<tr>
<td>Σ / 4</td>
<td></td>
<td></td>
<td></td>
<td>9.92</td>
<td>9.92</td>
<td>9.92</td>
<td>9.92</td>
<td>0.287</td>
</tr>
</tbody>
</table>

Resume: The best value of \( Y = 4.7 \) gives the combination (-1;1).

**Step I: Test of homogeneity of variance**

\( Y \) and \( s_i^2 \) (Table 3) are required to make the tests of homogeneity of variance, an important test to check the experiment design on the whole. The main idea is that the variances of the results are the same of each combination because only the factor levels were changed during the experiments (Formulas 3 and 4).

\[
=> \text{If this thesis } (H_0) \text{ is true, the experiment design is true, too, and next the quantities can be analysed.}
\]

\[
=> \text{If this thesis } (H_0) \text{ is false then may be the experiment design is false too (e.g. we forgot important factors) or the conditions and so other factors were not constant during the}
\]
experiment phases (e.g. we have had not the right conditions). Otherwise more experiments are perhaps necessary.

\[ H_0: s^2 = s_1^2 = s_2^2 = s_3^2 = s_4^2 \]  
(3)

If

\[ \frac{s_{\text{max}}}{s_{\text{min}}} \leq F_{f_1,f_2,95\%} \]  
(4)

then \( H_0 \) is true else \( H_0 \) is false.

For our example the values are

\[ f_1 = n - 1 = 3 - 1 = 2 \]  
(5)

\[ f_2 = n - 1 = 3 - 1 = 2 \]  
(6)

\[ F_{2,2,\text{calc}} \leq F_{2,2,95\%} \]  
(7)

that is \( 2.53 \leq 19 \) then \( H_0 \) is true.

The result of the test is (Compare formulas 5 – 7): Because the thesis \( H_0 \) is true the experiment design is true too and now should be analysed the quantities.

**Step II: Test of significance of each effect**

The constant values and the coefficients of the formula as following (Table 4 and formulas 8 -11) are calculated now:

\[ b_0 = \frac{\Sigma Y}{m} = 9.92 \]  
(8)

\[ b_1 = \frac{\Sigma x_1 Y}{m} = 3.25 \text{ significant} \]  
(9)

\[ b_2 = \frac{\Sigma x_2 Y}{m} = -2.15 \text{ significant} \]  
(10)

\[ b_{12} = \frac{\Sigma x_1 x_2 Y}{m} = -0.15 \text{ not significant} \]  
(11)

Table 4. Calculation of the constant values and the coefficients of the formula

<table>
<thead>
<tr>
<th>Combination</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 x_2 )</th>
<th>( Y )</th>
<th>( x_1 Y )</th>
<th>( x_2 Y )</th>
<th>( x_1 x_2 Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>8.7</td>
<td>-8.7</td>
<td>-8.7</td>
<td>8.7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>15.5</td>
<td>15.5</td>
<td>-15.5</td>
<td>-15.5</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>4.7</td>
<td>-4.7</td>
<td>4.7</td>
<td>-4.7</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10.9</td>
<td>10.9</td>
<td>10.9</td>
<td>10.9</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td></td>
<td></td>
<td></td>
<td>39.67</td>
<td>13.00</td>
<td>-8.60</td>
<td>-0.60</td>
</tr>
<tr>
<td>( \Sigma / 4 )</td>
<td></td>
<td></td>
<td></td>
<td>9.92</td>
<td>3.25</td>
<td>-2.15</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

The result is the formula 13 (compare to the common form of formula 12):

\[ y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2 \]  
(12)

\[ y = 9.92 + 3.25 x_1 - 2.15 x_2 - 0.15 x_1 x_2 \]  
(13)

Now the value \( t_{f,95\%} S_d \) is calculated to decide about the significance of the coefficients (formula 14):

\[ t_{f,95\%} S_d = 0.3046 \]  
(14)

The test gives the following results:

- The factor that most influence the results is the speed (3.25). A higher speed induces to a higher rate of failures.
- The factor that smaller influences the results is the pitch (2.15), but it is also significant. A smaller pitch induces to a higher rate of failures.
The existence of interactions and synergies between both factors (-0.15) is given, but it is not significant for the output, because the coefficient (-0.15) is smaller than the significance value 0.3046. Now it is possible to simplify the function as following (Formula 15 compare to formula 13):

\[ y_p = 9.92 + 3.25x_1 - 2.15x_2 \]  

The end of the experiments is not achieved. The task is now to further optimize speed and pitch. Therefore are new experiments necessary.

**Step III: Adapt the model**

The third test compares the formula and the results of the experiments (Table 5). The values for \( y_p \) are calculated by using formula 15. If the variability of the model is smaller than the experimental standard deviation, then the model can be accepted and further be used. (Formulas 16 – 20)

\[ F_{f_1, f_2, \text{Model}} \leq F_{f_1, f_2, 95\%} \quad (16) \]

\[ f_1 = m - (k + 1) = 4 - (2+1) = 1 \quad (17) \]

\[ f_2 = m = 4 \quad (18) \]

**Table 5. Calculation of the differences between calculated and experimental values**

<table>
<thead>
<tr>
<th>Combination</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1x_2 )</th>
<th>( Y )</th>
<th>( y_p )</th>
<th>( (Y-y_p)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>8.7</td>
<td>8.8</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>15.5</td>
<td>15.3</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>4.7</td>
<td>4.5</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10.9</td>
<td>11.0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

\[ \Sigma 39.67 \quad 39.67 \quad 0.08 \]

\[ \Sigma / 4 9.92 \quad 9.92 \quad 0.02 \]

\[ F_{1,4,\text{Model}} \leq F_{1,4,95\%} \quad (19) \]

\[ 0.08 \leq 7.71 \quad (20) \]

Because of the fact that the variability of the model is smaller than the experimental standard deviation the model can be accepted and further be used. In table 6 are listed the current results of the experiments.

**Table 6. Results of the experiments**

<table>
<thead>
<tr>
<th>Questions</th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>- How much experiments are necessary to analyse the problem?</td>
<td>8 (in this paper 12)</td>
</tr>
<tr>
<td>- How much failures will appear at each combination?</td>
<td>table 3</td>
</tr>
<tr>
<td>- How big is the influence of speed and of pitch on the rate of failures?</td>
<td>formula 13</td>
</tr>
<tr>
<td>- Is there any significant interaction between both factors – speed and pitch?</td>
<td>no</td>
</tr>
<tr>
<td>- Which combination of speed and pitch gets the lowest rate of failures?</td>
<td>low speed and high pitch</td>
</tr>
<tr>
<td>- Are there limits of the sorter visible?</td>
<td>no</td>
</tr>
</tbody>
</table>
4. SIX SIGMA AND LOGISTICS

At first Six Sigma is a methodology for eliminating defects. This is an important task in logistics too. But Six Sigma is also a philosophy to work and a business strategy too. The goal is to achieve a process level in standard deviations six sigma. After Motorola in the 1980s were published a lot of success stories, also in manufacturing, than in different services, in research, in healthcare and e.g. in government too. The list of enterprises which adapt six Sigma is important e.g. Xerox, Boeing, GE, Kodak, Sony, Polaroid, NASA, Dupont, Toshiba, Ford, ABB. The original terminology “Six Sigma” is based on the established statistical approach. This is described by a sigma measurement scale. The range is normally from one / two to six sigma. It defines how much of a product or process normal distribution is contained inside the specification.

Essentially, the higher the sigma value the less is the defect rate, because the process distribution is contained inside the specification. Some examples should show what this quality level Six Sigma means and why the traditional sigma level is not sufficient sometimes. Figure 4 shows the distribution of the quality for $C_p=C_{pk}=1$. There is to be mentioned that the limits have an area of uncertainty around them because of the measurement methods, the environment, human influences and other ones.

Very interesting are the values of the defects per million opportunities in table 7. A four sigma level means that there are allowed 6210 defects per million opportunities. The table 8 gives some examples from the world of logistics.

Table 7. Defects per million according to the sigma level and the uncertainty

<table>
<thead>
<tr>
<th>$C_p$</th>
<th>$C_{pk}$</th>
<th>Sigma</th>
<th>Inside the tolerance in %</th>
<th>Outside the tolerance /mil.</th>
<th>defects/mil. opportunities DPMO</th>
<th>defects / mil. with $u=0.01$ T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>-0.17</td>
<td>1</td>
<td>30.2328</td>
<td>302327.9</td>
<td>697672.1</td>
<td>695125.8</td>
</tr>
<tr>
<td>0.67</td>
<td>0.17</td>
<td>2</td>
<td>69.1230</td>
<td>691229.8</td>
<td>308770.2</td>
<td>312297.9</td>
</tr>
<tr>
<td>1.00</td>
<td>0.50</td>
<td>3</td>
<td>93.3189</td>
<td>933189.4</td>
<td>66810.6</td>
<td>70706.4</td>
</tr>
<tr>
<td>1.33</td>
<td>0.83</td>
<td>4</td>
<td>99.3790</td>
<td>993790.3</td>
<td>6209.7</td>
<td>7123.4</td>
</tr>
<tr>
<td>1.67</td>
<td>1.17</td>
<td>5</td>
<td>99.9767</td>
<td>999767.3</td>
<td>232.7</td>
<td>300.3</td>
</tr>
<tr>
<td>2.00</td>
<td>1.50</td>
<td>6</td>
<td>99.9997</td>
<td>9999996.6</td>
<td>3.4</td>
<td>5.1</td>
</tr>
</tbody>
</table>

The process capability can be now quantified by $c_p$, $c_{pm}$, $c_{pk}$, $c_{pnnk}$ or $c_{pnnkx}$ – index. For more information see [13]. After checking the values it is necessary to act to improve the results. There are only mentioned two methods (DoE and Six Sigma) of quality management in logistics in this paper. Please, use [14] for further information about other methods.
Table 8. Examples of the logistics: the traditional vs. the Six-Sigma view

<table>
<thead>
<tr>
<th>Examples of the logistics</th>
<th>Classical view</th>
<th>Six sigma view</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 millions containers are on tour worldwide at each moment [9]</td>
<td>712340 containers are missed</td>
<td>510 containers are missed</td>
</tr>
<tr>
<td>12 000 post-offices exist in Germany [10]</td>
<td>74 offices are closed</td>
<td>no office is closed</td>
</tr>
<tr>
<td>1,5 million customers were delivered with letters each day [10]</td>
<td>9315 customers get no or a wrong letter</td>
<td>8 customers get no or a wrong letter</td>
</tr>
<tr>
<td>There are in summary 12249 motor-vehicles for airtraffic in Germany in 2006 [11]</td>
<td>76 accidents with a motorplane</td>
<td>No accident with a motorplane</td>
</tr>
</tbody>
</table>
